Linear Algebra I

18/12/2017, Monday, 15:00 – 17:00

You are **NOT** allowed to use any type of calculators.

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1 $(2 + 7 + 3 + 5 + 3 = 20 \text{ pts})$ **Linear systems of equations**

Consider the following linear system of equations in the unknowns x, y, and z:

$$\begin{aligned} ax + y + z &= 1 \\ x + ay + z &= 1 \\ x + y + az &= 1. \end{aligned}$$

(a) Write down the augmented matrix.

(b) By performing elementary row operations, put the augmented matrix into row echelon form.

(c) Determine all values of a so that the system is inconsistent.

(d) Determine all values of a so that the system is consistent and find the solution set for such values of a.

(e) Determine all values of a so that the system has a unique solution.

2 (20 pts) **Determinants**

Find all values of a, b, c, d, e, and f such that the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ a & b & b & b \\ a & c & d & d \\ a & c & e & f \end{bmatrix}$$

is singular.

3 $(10 + 10 = 20$ pts) **Partitioned matrices**

Let A and B be $n \times n$ matrices. Suppose that A is nonsingular.

(a) Show that the matrix

$$M = \begin{bmatrix} A & B \\ B & A \end{bmatrix}$$

is nonsingular if and only if the matrix $A - BA^{-1}B$ is nonsingular.

(b) Suppose that $A - BA^{-1}B$ is nonsingular and find the inverse of M.

4 $(8 + 7 + 7 + 8 = 30$ pts) **Vector spaces**

(a) Let $E = (\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n)$ be an ordered basis for the vector space V.

(i) Show that the vectors $\boldsymbol{v}_1 + \boldsymbol{v}_2, \boldsymbol{v}_2 + \boldsymbol{v}_3, \ldots, \boldsymbol{v}_{n-1} + \boldsymbol{v}_n, \boldsymbol{v}_n$ form a basis for V.

(ii) Find the transition matrix corresponding to the change of basis from $E = (\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n)$ to $F = (\boldsymbol{v}_1 + \boldsymbol{v}_2, \boldsymbol{v}_2 + \boldsymbol{v}_3, \ldots, \boldsymbol{v}_{n-1} + \boldsymbol{v}_n, \boldsymbol{v}_n)$.

(b) Consider the vector space P_4. Let

$$S = \{p \in P_4 \mid p(1) = 0 \text{ and } p'(1) = 0\}$$

where $p'(x)$ denotes the derivative of $p(x)$.

(i) Show that S is a subspace.

(ii) Find a basis for S and determine its dimension.

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10 pts free